STRUCTURE OF EQUILIBRIUM PSEUDOTURBULENCE IN GASEOUS SUSPENSIONS IN CONDITIONS OF LOCAL NONHOMOGENEITY

Yu. A. Buevich and V. G. Markov

In streams of dispersed systems there are intense pulsations of the particles and of the fluid phase. Such pulsative motion (termed hereafter "pseudoturbulence") usually has a definitive influence on the formation of the rheological properties of the dispersed systems and on the intensity of the transport processes taking place therein.

The mechanism of the occurrence of pseudoturbulent motions, associated with the work of the external field forces, and of the viscous phase interaction forces on the fluctuations of the dispersed system concentration, is discussed in detail, for example, in [1] for the example of a fluidized bed. Phenomenologically, the presence of pseudoturbulence is expressed in deviation of the true instantaneous local values of the velocities of the fluid and the particles, of the bulk concentration of the system, and of the pressure from the corresponding mean values $\mathbf{v}, \mathbf{w}$, ρ, and p (termed hereafter the "dynamic variables"). The hydrodynamic model of an arbitrary dispersed system was proposed in [2]. In the following the results of [2] are applied to the study of steady pseudoturbulence in an infinite stream of a gaseous suspension, provided the gradients of all the dynamic variables other than the pressure p are negligibly small. We term such pseudoturbulence "equilibrium." In this study we examine only the "nonhomogeneous" regime, when there is aggregation of the particles in the system. Specifically, the mean square velocities of the phase pulsations in this regime, the effective diffusion coefficients in different directions, and other pseudoturbulence characteristics are calculated. The dimensions of the nonhomogeneities which develop are estimated and a criterion for onset of the nonhomogeneous pseudoturbulence regime is obtained.

1. Spectral Measures and Densities. We examine in the following the deviations $\mathbf{v}'$, $\mathbf{w}'$, ρ', p' as random functions of the coordinates and time. Any such deviation can be represented in the form of the Fourier–Stieltjes stochastic integral, for example,

$$\rho'(t, \mathbf{r}) = \int e^{i(\omega t + \mathbf{kr})}\, dZ_\rho$$

where dZ_ρ is the spectral measure of the random process $\rho'(t, \mathbf{r})$. The properties of the spectral measures and the rules for calculating the various correlation functions of the random processes in question are described, for example, in [3]. The equations for the spectral measures dZ_v, dZ_w, dZ_ρ, dZ_p were obtained and solved in [2]. Their solution for the case of a gaseous suspension, when gravity, momentum, and the viscous stresses in the gas can be neglected have the form

$$dZ_p = -\frac{i\omega d_2 \beta K \rho}{[i\omega(1-\rho) + \beta K]\,k^2}\left[\frac{\omega}{1-\rho} + \left(\frac{1}{1-\rho} + \frac{d\ln K}{dp}\right)\mathbf{uk}\right] dZ_\rho$$

$$d\mathbf{Z}_v = \left\{\left[\frac{\omega}{1-\rho} + \left(\frac{1}{1-\rho} + \frac{d\ln K}{d\rho}\right)\mathbf{uk}\right]\frac{\mathbf{k}}{k^2} - \frac{d\ln K}{d\rho}\mathbf{u}\right\} dZ_\rho \tag{1.1}$$

$$d\mathbf{Z}_w = \frac{\beta K}{i\omega(1-\rho) + \beta K}\left[\frac{\omega}{1-\rho} + \left(\frac{1}{1-\rho} + \frac{d\ln K}{d\rho}\right)\mathbf{uk}\right]\frac{\mathbf{k}}{k^2}\, dZ_\rho$$

$$\beta = \frac{9\varkappa\nu_0}{2a^2}, \quad \nu_0 = \frac{\mu_0}{d_1}, \quad \varkappa = \frac{d_1}{d_2}, \quad \mathbf{u} = \mathbf{v} - \mathbf{w}$$

Here d_1 and d_2 are the densities of the gas and the particle material, a is the particle radius, μ_0 is the gas viscosity, and $K = K(\rho)$ $[K(0) = 1]$ is a function indicating the factor by which the viscous force

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acting from the carrying gas stream on the particle in the concentrated gaseous suspension, in which the relative position of the particles is constant, increases in comparison with the Stokes force acting on an isolated particle.

Relations (1.1) make it possible to express the spectral densities $\Psi_{\varphi,\psi}(\omega, \mathbf{k})$ of the arbitrary random processes $\varphi'(t, \mathbf{r})$ and $\psi'(t, \mathbf{r})$ in the form of functions of the dynamic variables and the spectral density $\Psi_{\rho,\rho}(\omega, \mathbf{k})$ of the random process $\rho'(t, \mathbf{r})$. We have the representations

$$\Psi_{vi,vj}(\omega, \mathbf{k}) = \frac{1}{(1-\rho)^2}\Big[(\omega + cuk_1)^2\frac{k_ik_j}{k^4} - (c-1)\,u\,(\omega + cuk_1)\times \frac{\delta_{i1}k_j + \delta_{j1}k_i}{k^2} + (c-1)^2u^2\delta_{i1}\delta_{j1}\Big]\Psi_{\rho,\rho}(\omega, \mathbf{k}), \quad c = 1 + (1-\rho)\frac{d\ln K}{d\rho}$$

$$\Psi_{wi,wj}(\omega, \mathbf{k}) = \left(\frac{\omega_0}{1-\rho}\right)^2\frac{(\omega + cuk_1)^2}{\omega^2 + \omega_0^2}\frac{k_ik_j}{k^4}\Psi_{\rho,\rho}(\omega, \mathbf{k}), \quad \omega_0 = \frac{\beta K}{1-\rho} \tag{1.2}$$

$$\Psi_{\rho,vj}(\omega, \mathbf{k}) = \frac{1}{1-\rho}\Big[(\omega + cuk_1)\frac{k_j}{k^2} - (c-1)\,u\delta_{j1}\Big]\Psi_{\rho,\rho}(\omega, \mathbf{k})$$

$$\Psi_{\rho,wj}(\omega, \mathbf{k}) = \frac{\omega_0}{1-\rho}\frac{-i\omega + \omega_0}{\omega^2 + \omega_0^2}(\omega + cuk_1)\frac{k_j}{k^2}\Psi_{\rho,\rho}(\omega, \mathbf{k})$$

$$\Psi_{vi,wj}(\omega, \mathbf{k}) = \frac{\omega_0}{(1-\rho)^2}\frac{-i\omega + \omega_0}{\omega^2 + \omega_0^2}\Big[(\omega + cuk_1)^2\frac{k_ik_j}{k^4} - (c-1)\,u\,(\omega + cuk_1)\frac{k_j\delta_{i1}}{k^2}\Big]\Psi_{\rho,\rho}(\omega, \mathbf{k}), \quad \Psi_{\varphi,\psi}(\omega, \mathbf{k}) = \langle dZ_\varphi^* dZ_\psi\rangle$$

Here the x_1 coordinate axis is taken in the direction of the interphase slip vector $\mathbf{u}$.

The following expression for the spectral density $\Psi_{\rho,\rho}(\omega, \mathbf{k})$ was obtained in [2]

$$\Psi_{\rho,\rho}(\omega, \mathbf{k}) = \frac{A}{\pi}\frac{\Phi_{\rho,\rho}(\mathbf{k})}{\omega^2 + (A - B\omega^2)^2}, \quad B = \frac{\operatorname{tr}\mathbf{D}}{\langle w'^2\rangle} = \frac{D_1 + 2D_2}{\langle w'^2\rangle}, \quad A = \mathbf{kDk} = D_1k_1^2 + D_2(k_2^2 + k_3^2) \tag{1.3}$$

Here $\mathbf{D}$ is the particle pseudoturbulent diffusivity tensor with the eigenvalues D_1, D_2, $D_3 = D_2$, and $\Phi_{\rho,\rho}(\mathbf{k})$ is the spectral density of the quantity ρ', examined at a fixed moment of time, i.e., as a random function of the coordinates only.

If the particles were points, i.e., if their positions could be defined to within the δ-functions, then $\Phi_{\rho,\rho}(\mathbf{k}) = \text{const}$ would follow from the equivalence of the particles and the statistical homogeneity of the space (see, for example, [4]). In reality the positions of the centers of the particles are defined to within some volume σ_f. Accounting for this circumstance with the aid of the procedure for smoothing the shortwave region of the concentration fluctuation spectrum, proposed by Massignon [5], made it possible to obtain in [6] the expression

$$\Phi_{\rho,\rho}(\mathbf{k}) = \frac{3\sigma_0}{8\pi^3}\rho\left(1 - \frac{\rho}{\rho_*}\right)\frac{\sin kb_f - kb_f\cos kb_f}{(kb_f)^3} \quad \begin{pmatrix}\sigma_0 = {}^4/_3\pi a^3\\ \sigma_f = {}^4/_3\pi b^3\end{pmatrix} \tag{1.4}$$

where ρ_* is the concentration of the gaseous suspension in the dense packing state. It was assumed in [6] that the volume σ_f is simply equal to the mean particle specific volume $\sigma = \sigma_0\rho^{-1}$. Thereby no account was taken for the effects of screening the displacements of some particle in its specific volume by the neighboring particles, which led to the necessity to account for this screening separately, for example, in calculating the effective diffusion coefficients of particles in concentrated systems. Here we shall use as the measure of the volume σ_f the free volume $\sigma_0 - \sigma_*$, where $\sigma_* = \sigma_0\rho_*^{-1}$, as is usually done in statistical physics of fluids. Then we have

$$b_f = a\rho^{-1/3}(1 - \rho/\rho_*)^{1/3} = b\,(1 - \rho/\rho_*)^{1/3}$$

The function $\Phi_{\rho,\rho}(\mathbf{k})$ in the form (1.4) is somewhat inconvenient for the subsequent calculations. Therefore we replace it approximately by a step-function such that the integral of $\Phi_{\rho,\rho}(\mathbf{k})$ over wave space remains as before; we obtain after calculations

$$\Phi_{\rho,\rho}(\mathbf{k}) = \frac{3}{4\pi}\frac{\rho^2}{k_0^3}\left(1 - \frac{\rho}{\rho_*}\right)Y(k_0 - k), \quad k_0 = k_\infty = \left(\frac{3\pi\rho}{2}\right)^{1/3}\cdot\left(1 - \frac{\rho}{\rho_*}\right)^{-1/3}\frac{1}{a} \tag{1.5}$$

Here $Y(x)$ is the Heaviside function. We note that $\Phi_{\rho,\rho}(\mathbf{k})$ in the form (1.5) can also be obtained directly if we use in place of the Massignon method the well known Debye method, which is essentially the Massignon method applied in wave space rather than in real space (see, for example, [1]). We also note that to within a constant, the cofactors of (1.4) and (1.5) are Fourier transforms of one another.

If the particles cannot be considered statistically independent and in the gas suspension flow there are correlated motions of entire particle groups ("packets"), the formula (1.5) for $\Phi_{\rho,\rho}(\mathbf{k})$ can as before be considered valid, but $k_0 < k_{\infty}$. In this case the decrease of k_0 reflects simply the reduction of the number of degrees of freedom of the system of particles with the appearance of the correlations between the particles and the corresponding decrease of the number of harmonics in the Fourier representation of the gaseous suspension concentration fluctuations. The equation for finding k_0 is examined later.

Integrating the spectral densities (1.2) with respect to the frequency ω and using (1.3), we obtain the expressions for the partial spectral densities

$$\Phi_{vi,vj}(\mathbf{k}) = \frac{\Phi_{\rho,\rho}(\mathbf{k})}{(1-\rho)^2}\left[\left(\frac{A}{B} + c^2u^2k_1^2\right)\frac{k_ik_j}{k^4} - c(c-1)u^2\frac{k_1(\delta_{i1}k_j + \quad i}{k^2} + (c-1)^2u^2\delta_{i1}\delta_{j1}\right],$$

$$\Phi_{wi,wj}(\mathbf{k}) = \frac{\omega_0^2\Phi_{\rho,\rho}(\mathbf{k})}{(1-\rho)^2}\frac{A}{A+\omega_0+B\omega_0^2}\left(1 + c^2u^2k_1^2\frac{1+B\omega_0}{A\omega_0}\right)\frac{k_ik_j}{k^4},$$

$$\Phi_{\rho,vj}(\mathbf{k}) = \frac{\Phi_{\rho,\rho}(\mathbf{k})}{1-\rho}\left[c\frac{k_1k_j}{k^2} - (c-1)\delta_{j1}\right]u, \tag{1.6}$$

$$\Phi_{\rho,wj}(\mathbf{k}) = \frac{\omega_0\Phi_{\rho,\rho}(\mathbf{k})}{1-\rho}\frac{A}{A+\omega_0+B\omega_0^2}\left(\frac{1}{\omega_0} + cuk_1\frac{1+B\omega_0}{A\omega_0}\right)\frac{k_j}{k^2}$$

$$\Phi_{vi,wj}(\mathbf{k}) = \frac{\omega_0^2\Phi_{\rho,\rho}(\mathbf{k})}{(1-\rho)^2}\frac{A}{A+\omega_0+B\omega_0^2}\left\{\left[(1+c^2k^2{}_1u^2)\frac{k_ik_j}{k^4}\right.\right.$$

$$\left.\left. - c(c-1)u^2\frac{k_1k_j\delta_{i1}}{k^2}\right]\frac{1+B\omega_0}{A\omega_0} - i\left(2ck_1\frac{k_ik_j}{k^4} - (c-1)\frac{k_j\delta_{i1}}{k^2}\right)\frac{u}{\omega_0}\frac{1+B\omega_0}{A\omega_0}\right\}$$

The relations (1.6) contain the quantity B, which in accordance with (1.3) depends on the unknowns $\langle w'^2\rangle$, D_1, D_2. For the sequel it is necessary to find explicit representations for these unknowns in terms of the dynamic variables.

2. Characteristics of Equilibrium Pseudoturbulence. We use the formula for finding D_1, D_2

$$D_j \approx \int_0^\infty d\tau \iint e^{i\omega\tau}\Psi_{wj,wj}(\omega,\mathbf{k})\,d\omega d\mathbf{k} = \int_0^\infty R_{wj,wj}(\tau,0)\,d\tau, \quad R_{wi,wj}(\tau,\xi) = \langle w_i'(t,\mathbf{r})\,w_j'(t+\tau,\mathbf{r}+\xi)\rangle \tag{2.1}$$

(summation over j is not performed here). Using the relations (1.2), (1.3), (1.5) for the integration in (2.1) and changing the order of integration with respect to ω and τ, we obtain the equations

$$D_1 = \frac{2\pi c^2u^2}{(1-\rho)^2k_0^2}\frac{\Phi}{D_1-D_2}\left(\frac{1}{3} - r^2 + r^2\operatorname{arc\,tg}\frac{1}{r}\right), \quad \Phi = \frac{3}{4\pi}\rho^2\left(1-\frac{\rho}{\rho_*}\right)$$

$$D_2 = \frac{\pi c^2u^2}{(1-\rho)^2k_0^2}\frac{\Phi}{D_1-D_2}\left(\frac{2}{3} + r^2 - r(1+r^2)\operatorname{arc\,tg}\frac{1}{r}\right), \quad r^2 = \frac{D_2}{D_1-D_2} \tag{2.2}$$

From (2.2) we have the universal transcendental equation for r

$$\alpha = \frac{r^2}{1+r^2} = \frac{1 + {}^3/_2r^2 - {}^3/_2r(1+r^2)\operatorname{arc\,tg}(1/r)}{1-3r^2+3r^3\operatorname{arc\,tg}(1/r)}, \quad \alpha = \frac{D_2}{D_1} \tag{2.3}$$

It is not difficult to show that (2.3) has the single positive root $r \approx 0.855$; the corresponding value of $\alpha \approx 0.4221$. Thus the ratio of the particle pseudoturbulent diffusion coefficients in the longitudinal (along $\mathbf{u}$) and transverse directions is a universal constant, which is independent of both the dynamic variables and the physical parameters of the phases.

Solving (2.2) we have

$$D_1 = \frac{2cu}{(1-\rho)k_0}\left[\pi\Phi(1+r^2)\left(\frac{1}{3} - r^2 + r^3\operatorname{arc\,tg}\frac{1}{r}\right)\right]^{1/2}, \quad D_2 = 0.422D_1$$

Or, substituting herein r and α, we finally obtain

$$D_1 = \frac{0.859cu}{k_0}\frac{\rho}{1-\rho}\left(1-\frac{\rho}{\rho_*}\right)^{1/2}, \quad D_2 = 0.422D_1 \tag{2.4}$$

Similarly to (2.1), we have the equation for

$$\langle w'^2\rangle = \frac{\omega_0}{(1-\rho)^2}\int\frac{\Phi_{\rho,\rho}(\mathbf{k})}{k^2}\cdot\frac{A\omega_0 + c^2u^2k_1^2(1+B\omega_0)}{A+\omega_0(1+B\omega_0)}\,d\mathbf{k} \tag{2.5}$$

TABLE 1. Dependence of Ω, Δ, X_j ($j = 1, 2, 4, 6, 8$) on ρ*

ρ	Ω	Δ	$10^3 X_1$	$10^3 X_2$	$10^3 X_4$	$10^3 X_6$	X_8
0.05	0.2329	1.5968	0.2917	0.1059	0.1694	0.4782	99.243
0.10	0.2513	1.6596	1.2294	0.4444	0.6880	1.9037	45.690
0.15	0.2721	1.7293	2.9041	1.0464	1.5684	4.2332	28.006
0.20	0.2958	1.8073	5.3930	1.9392	2.8153	7.3710	19.290
0.25	0.3229	1.8956	8.7397	3.1357	4.4155	11.144	14.167
0.30	0.3542	1.9964	12.906	4.6243	6.3202	15.273	10.850
0.35	0.3906	2.1129	17.697	6.3360	8.4105	19.326	8.5777
0.40	0.4333	2.2497	22.613	8.0901	10.435	22.647	6.9837
0.45	0.4834	2.4142	26.577	9.4938	11.893	24.255	5.8890
0.50	0.5426	2.6184	27.439	9.7592	11.782	22.638	5.2578
0.55	0.6118	2.8830	21.085	7.4179	8.7336	15.736	5.3949

*For $\rho = 0$ we have $\Omega = 0.2165$, $\Delta = 1.5403$, $X_j = 0$; for $\rho = 0.6$, $\Omega = 0.6912$, $\Delta = 3.2481$, $X_j = 0$.

We obtain the equation for k_0 (for $k_0 < k_\infty$) by equating the dissipation ε_1 of pseudoturbulence energy by small-scale isotropic "vibrations" of the particles within the limits of their specific volumes to the viscous (Brownian) dissipation ε_2 of the energy of these vibrations by viscous forces. The same equation was used in [6] for somewhat different purposes. It is actually assumed that the dissipation of pseudoturbulent energy into heat is accomplished through the mentioned small-scale vibrations; a detailed discussion of this condition in connection with experimental data is presented in [7].

We use for ε_2 the usual expression, which follows from Brownian motion theory

$$\varepsilon_2 = 3\rho d_2 \beta^2 K^2 D_m,$$

where D_m is the coefficient of diffusion owing to the considered vibrations.

We represent the quantity ε_1 is the form which is usual in hydrodynamics of a viscous fluid with the effective viscosity $\mu_m = \rho d_2 D_m$. This may be done, strictly speaking, only if the pseudoturbulence linear scale is much longer than the average distance between the particles in the system, i.e., $k_0 \ll k_\infty$. This is usually the case for gaseous suspension flows. We then obtain similarly [7] the equation

$$3\beta^2 K^2 = \frac{\omega_0}{(1-\rho)^2} \int \Phi_{\rho,\rho}(\mathbf{k}) \frac{A\omega_0 + c^2 u^2 k_1^2 (1 + B\omega_0)}{A + \omega_0 (1 + B\omega_0)} d\mathbf{k} \tag{2.6}$$

Equation (2.6) differs from the same equation in [7] by the absence in the right side of the term owing to "compressibility" of the dispersed phase (by the nonzero divergence w') resulting from possible changes of the gaseous suspension concentration in the flow. This refinement is introduced because such "compressibility" is not accompanied by energy dissipation.

We introduce the dimensionless parameters

$$\Omega = \frac{\omega_0}{D_1 k_0^2}, \quad \Delta = (1 + 2\alpha) \frac{D_1 \omega_0}{\langle w'^2 \rangle}, \quad Z = \frac{D_1 k_0}{cu} \tag{2.7}$$

From (2.4) for D_1 we can write Z in the form

$$Z^2 = \frac{1}{\gamma} \left(\frac{\rho}{1-\rho} \right)^2 \left(1 - \frac{\rho}{\rho_*} \right), \qquad \gamma = 1.3563$$

Equations (2.5) and (2.6) are then written in the form

$$\frac{1 + 2\alpha}{\Delta} = 3\gamma \int_0^1 \int_0^1 \frac{\Omega Z^2 [t^2 + \alpha (1 - t^2)] + (1 + \Delta) t^2}{\xi^2 [t^2 + \alpha (1 - t^2)] + (1 + \Delta) \Omega} \xi^2 d\xi dt$$

$$\Omega = \frac{\gamma}{(1-\rho)^2} \int_0^1 \int_0^1 \frac{\Omega Z^2 [t^2 + \alpha (1 - t^2)] + (1 + \Delta) t^2}{\xi^2 [t^2 + \alpha (1 - t^2)] + (1 + \Delta) \Omega} \xi^4 d\xi dt \tag{2.8}$$

These equations were solved on the BESM computer for different ρ in the interval from zero to $\rho_* = 0.6$. The resulting relations are shown in Table 1. The values of the parameter Ω together with the expression (2.4) for D_1 define the quantity k_0. This k_0 is meaningful only for $k_0 < k_\infty$. If the solution k_0 of the system (2.5), (2.6) is larger than the k_∞ from (1.5), we must take $k_0 \equiv k_\infty$.

TABLE 2. Dependence of X_j (j = 3, 5, 7) on ρ and n*

ρ	n = 1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.05	0.2964	0.4065	0.5080	0.5944	0.6667	0.7275	0.7789
	0.0401	—0.0103	—0.0433	—0.0678	—0.0858	—0.0993	—0.1109
	—0.4020	—0.6433	—0.8041	—0.9190	—1.0051	—1.0721	—1.1257
0.10	1.2033	1.6491	2.0606	2.4106	2.7036	2.9497	3.1580
	0.1718	—0.0397	—0.1807	—0.2815	—0.3570	—0.4158	—0.4628
	—1.5432	—2.4691	—3.0364	—3.5273	—3.8580	—4.1152	—4.3210
0.15	2.7411	3.7532	4.6375	5.4819	6.1473	6.7059	7.1788
	0.4140	—0.0840	—0.4160	—0.6532	—0.8311	—0.9694	—1.0801
	—3.3088	—5.2941	—6.6176	—7.5630	—8.2721	—8.8235	—9.2647
0.20	4.9148	6.7203	8.3870	9.8042	10.991	11.988	12.831
	0.7862	—0.1352	—0.7494	—1.1882	—1.5173	—1.7732	—1.9779
	—5.5556	—8.8889	—10.111	—12.693	—13.889	—14.815	—15.556
0.25	7.6969	10.506	13.093	15.303	17.149	18.699	20.012
	1.3105	—0.1754	—1.1659	—1.8735	—2.4041	—2.8169	—3.1471
	—8.1019	—12.963	—16.204	—18.519	—20.255	—21.605	—22.635
0.30	10.997	14.977	18.650	21.774	24.390	26.537	28.446
	1.9965	—0.1854	—1.6400	—2.6791	—3.4583	—4.0644	—4.5493
	—10.714	—17.143	—21.429	—24.490	—26.786	—28.571	—30.000
0.35	14.604	19.839	24.672	28.781	32.222	35.112	37.558
	2.8303	—0.1430	—2.1252	—3.5410	—4.6029	—5.4288	—6.0896
	—13.088	—20.940	—26.175	—29.915	—32.719	—34.900	—36.645
0.40	18.083	24.502	30.428	35.467	39.638	43.231	46.221
	3.7400	0.0344	—2.5508	—4.3482	—5.6962	—6.7447	—7.5834
	—14.815	—23.704	—29.630	—33.862	—37.037	—39.506	—41.481
0.45	20.584	27.837	34.531	40.223	44.990	48.994	52.382
	4.5261	0.1161	—2.8240	—4.9240	—6.4991	—7.7241	—8.7041
	—15.341	—24.545	—30.632	—35.065	—38.352	—40.909	—42.995
0.50	20.532	27.754	34.421	40.090	44.838	48.824	52.199
	4.7513	0.2137	—2.8113	—4.9721	—6.5993	—7.8531	—8.8614
	—13.839	—22.222	—27.778	—31.746	—34.722	—37.037	—38.889
0.55	15.145	20.539	25.519	29.753	33.299	36.277	38.797
	3.6014	0.1046	—2.2265	—3.8917	—5.1405	—6.1119	—6.8839
	—9.3364	—14.938	—18.673	—21.340	—23.341	—24.897	—26.142

*The first numbers in the table groups give $10^3 X_3$, the second are $10^3 X_5$, and the third are $10^3 X_7$.

In the following we examine only the "nonhomogeneous" flow regimes of a gaseous suspension, when in the flows there are formed "packets" of particles moving together, cavities containing pure gas, and so on [8], where in view of the existence of correlations between the behavior of neighboring particles $k_0 < k_\infty$. Analysis of homogeneous flows, in which the particles can be considered statistically independent and $k_0 = k_\infty$ differs essentially from that presented here, requires separate examination. In this case we have the first equation of the system (2.8) and the relation $k_0 = k_\infty$ in place of the second equation.

Using the expressions for the spectral densities from section 1, we have

$$\langle v_i'p'\rangle = \langle w_i'p'\rangle = \langle \rho'p'\rangle = \langle v_1'v_2'\rangle = \langle w_1'w_2'\rangle = \langle v_1'w_2'\rangle = \langle v_2'w_1'\rangle = \langle \rho'v_2'\rangle = \langle \rho'w_2'\rangle = 0$$

The nonzero averages are written in the form

$$X_1 = \frac{\langle w_1'^2\rangle}{c^2u^2} = \frac{3\gamma\Omega Z^2}{1-\alpha}\{\Omega Z^2[\alpha J_2 + (1-\alpha)J_4] + (1+\Delta)J_4\} \tag{2.9}$$

$$X_2 = \frac{\langle w_2'^2\rangle}{c^2u^2} = \frac{3\gamma\Omega Z^2}{2(1-\alpha)}\{\Omega Z^2[\alpha J_0 + (1-2\alpha)J_2 - (1-\alpha)J_4] + (1+\Delta)(J_2 - J_4)\}, \quad \langle w_2'^2\rangle \equiv \langle v_2'w_2'\rangle$$

$$X_3 = \frac{\langle v_1'^2\rangle}{c^2u^2} = \gamma Z^2\left[\frac{1}{15}\frac{3+2\alpha}{1+2\alpha}X_0 + \frac{1}{5} - \frac{2}{3}\frac{c-1}{c} + \left(\frac{c-1}{c}\right)^2\right]$$

$$X_4 = \frac{\langle v_2'^2\rangle}{c^2u^2} = \frac{\gamma Z^2}{15}\left(\frac{1+4\alpha}{1+2\alpha}X_0 + 1\right), \quad X_0 = \frac{\Omega Z^2(1+2\alpha)}{\Delta}$$

$$X_5 = \frac{3\gamma\Omega Z^2}{1-\alpha}\left\{\Omega Z^2[\alpha J_2 + (1-\alpha)J_4] + (1+\Delta)\left(J_4 - \frac{c-1}{c}J_2\right)\right\}$$

$$X_6 = \frac{3\gamma\Omega Z^2}{1-\alpha}(1+\Delta)(1-\rho)J_2, \quad X_5 = \frac{\langle v_1'w_1'\rangle}{c^2u^2}, \quad X_6 = \frac{\langle \rho'w_1'\rangle}{cu}$$

$$X_7 = \frac{\langle \rho'v'_1\rangle}{cu} = \gamma Z^2(1-\rho)\left(\frac{1}{c} - \frac{2}{3}\right), \quad X_8 = \frac{cuk_0}{\omega_0} = \frac{1}{\Omega Z}$$

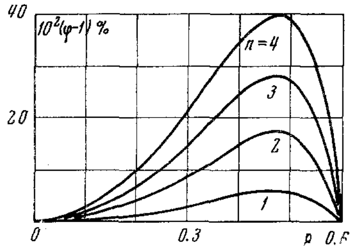

Fig. 1

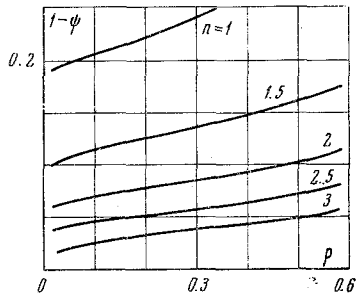

Fig. 2

$$J_m = \int_0^1\int_0^1 \frac{t^m dt d\xi}{t^2 + \tau^2(\xi)}, \quad \tau^2(\xi) = \frac{\alpha\xi^2 + \Omega(1+\Delta)}{(1-\alpha)\xi^2}$$

$$J_0 = \frac{1-\alpha}{\alpha}(Y_1 - Y_2), \quad J_2 = \left(\Omega Z^2 + \frac{1+\Delta}{1-\alpha}\right)^{-1}\left(\frac{1+2\alpha}{3\gamma\Delta} - \frac{\alpha}{1-\alpha}\Omega Z^2 J_0\right)$$

$$J_4 = \frac{1}{3} - \frac{3}{2}\frac{\alpha}{1-\alpha}J_2 + \frac{1}{2}\frac{\alpha+\Omega(1+\Delta)}{1-\alpha}(1-Y_1) - \frac{1}{2}Y_2$$

$$Y_1 = \left(\frac{\alpha\Omega(1+\Delta)}{1-\alpha}\right)^{1/2} \operatorname{arc\,tg}\left(\frac{1-\alpha}{\alpha\Omega(1+\Delta)}\right)^{1/2}, \quad Y_2 = (\Omega(1+\Delta))^{1/2}\operatorname{arctg}(\Omega(1+\Delta))^{-1/2}$$

The concrete calculations of the parameters (2.9) were also made on a computer, and it was always assumed that $\rho_* = 0.6$. The dependence of the quantities X_j (j = 1, 2, 4, 6, 8) on ρ are presented in Table 1. The quantities X_j (j = 3, 5, 7) depend on the form of the function $K(\rho)$. Experimental data on this function are presented in [9]; however there is no sufficiently convenient empirical formula for $K(\rho)$, suitable over the entire interval of variation of ρ for different values of the Reynolds and Archimedes numbers, in [9]. Here we shall use for simplicity an approximate formula of the form

$$K(\rho) = (1-\rho)^{-n}, \qquad c = 1 + n \tag{2.10}$$

The parameter n varies from 1-2 for large values of the Reynolds and Archimedes numbers to 3-4 for small values of these numbers. For example, according to [10] n varies from 1.39 to 3.65, according to [11] it varies from 1.375 to 3.75. However we note that the data of [10, 11] were obtained for a bed of particles in the fluidized state, i.e., containing pseudoturbulent motions. Nevertheless, according to [9] the values of the parameter n for a bed of motionless particles are close to the indicated values. In the following all the calculations are made for values of n from 1 to 4; the dependences of the parameters X_j (j = 3, 5, 7) for these n are shown in Table 2.

We introduce the ratios

$$N_v = \frac{\langle v_2'^2\rangle}{\langle v_1'^2\rangle} = \frac{X_4}{X_3}, \qquad N_w = \frac{\langle w_2'^2\rangle}{\langle w_1'^2\rangle} = \frac{X_2}{X_1}$$

It is easy to see that the first ratio depends on ρ and n, the second depends only on ρ, and the dependence on ρ is very weak, so that N_v and N_w can be approximated successfully by constants. For example, with variation of ρ from 0.05 to 0.55 the quantity N_w varies monotonically from 0.3630 to 0.3518, and N_v for n = 2 and n = 4 varies from 0.3335 to 0.3423 and from 0.2175 to 0.2251, respectively. We note that N_v decreases significantly with increase of n.

The total gas flux Q relative to the stationary dispersed phase equals the sum of the "regular" and "irregular" pseudoturbulent fluxes

$$Q = (1-\rho)u_Q = (1-\rho)u - \langle\rho' v_1'\rangle = (1-\rho)\varphi u, \quad \varphi = 1 - (1-\rho)^{-1} c X_7 \tag{2.11}$$

The quantity φ shows how many times the total flux Q exceeds the regular flux $(1-\rho)u$; its dependence on ρ for different n is shown in Fig. 1. In view of the negativity of X_7 in Table 2, the pseudoturbulent flux is positive and for sufficiently large n can amount to 30-40% of the regular flux. Relation (21) makes it possible to express all the pseudoturbulence characteristics from (2.9) in terms of the observable quantity – the total relative gas flux Q.

The quantity [2]

$$\psi = 1 - \frac{\langle \mathbf{v}'\mathbf{w}'\rangle^2}{\langle v'^2\rangle\langle w'^2\rangle} = 1 - \frac{(X_5 + 2X_2)}{(X_1 + 2X_2)(X_3 + 2X_4)}$$

is of interest for the pseudoturbulent energy transport equation under nonequilibrium conditions.

The dependence of ψ on ρ and n is illustrated in Fig. 2. The difference of ψ from one is significant only for small n, and for all n the quantity ψ can be approximated by a linear function of ρ.

Expressing k_0 through X_8 from (2.9) and using ω_0 and k_∞ from (1.2) and (1.5), we write the condition of local nonhomogeneity of the gaseous suspension in the form

$$k_0 = \frac{\beta K}{1-\rho}\frac{X_8}{cu}, \qquad \frac{\beta K a}{u} < \left(\frac{3\pi\rho}{2}\right)^{1/3}\left(1-\frac{\rho}{\rho_*}\right)^{-1/3}\frac{c(1-\rho)}{X_8} \tag{2.12}$$

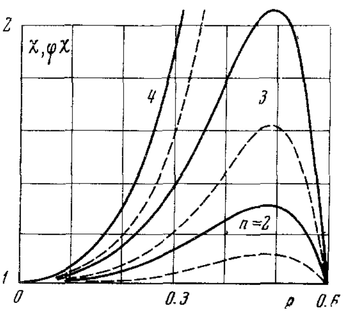

Fig. 3

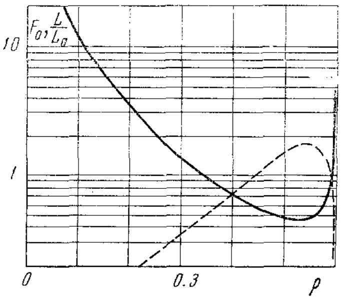

Fig. 4

A detailed discussion of this condition is presented below, but we see immediately from (2.12) that any gaseous suspension in sufficiently rarefied ($\rho \sim 0$) or nearly closely packed ($\rho \sim \rho_*$) states must be considered locally homogeneous, i.e., $k_0 = k_\infty$.

Thus all the pseudoturbulent averages are expressed through the dynamic variables. When necessary the same approach can be used to obtain the expressions for the various correlation functions as well.

Study of Equilibrium Pseudoturbulence and Its Influence on the Average Motion. In gradient-free stationary flow (i.e., under equilibrium conditions) the dynamic equations from [2] have the form

$$-\frac{1}{d_2}\frac{dp}{d\mathbf{r}} + \mathbf{g} + \beta K_1\mathbf{u} = 0, \qquad -\frac{1-\rho}{d_2}\frac{dp}{d\mathbf{r}} - \beta\rho K_1\mathbf{u} = 0$$
$$K_1 = \frac{K}{\chi} = K + c(X_7 - X_6)\frac{dK}{d\rho} + \frac{\rho^2}{2}\left(1-\frac{\rho}{\rho_*}\right)\frac{d^2K}{d\rho^2} \tag{3.1}$$

Here g is the external mass field acceleration vector.

The solution of (3.1) has the form

$$\frac{dp}{d\mathbf{r}} = \rho d_2 \mathbf{g}, \qquad \mathbf{u} = -\frac{(1-\rho)\mathbf{g}}{\beta K_1} = -\frac{\chi(1-\rho)\mathbf{g}}{\beta K} \tag{3.2}$$

The parameter χ shows how many times the viscous resistance of the motionless granular material exceeds the resistance of a bed of particles of the same porosity in a state of equilibrium pseudoturbulent motion for zero pseudoturbulent flux. Actually, in a motionless bed the gas velocity u^* equals

$$u^* = -\frac{(1-\rho)g}{\beta K}, \qquad \frac{u}{u_*} = \frac{K}{K_1} = \chi$$

The corresponding ratio of the total relative fluxes is

$$\frac{u_Q}{u^*} = \varphi\chi > 1 \tag{3.3}$$

This same quantity $\varphi\chi$ characterizes the ratio of the effective viscous resistances of motionless and moving beds of the same porosity for the same gas flux Q. Thus the pressure difference required to provide a given gas flux through a bed of randomly pulsating particles is less than the same difference for the bed of regularly packed motionless particles. This reduction of the effective resistance of particles entrained into the pseudoturbulent motion is the result of two factors which act in the same direction:

1) the appearance of the positive pseudoturbulent flux $\langle \rho' v_1' \rangle$, which increases the total gas flux for a fixed pressure gradient, and

2) the appearance of a negative fluctuational addition to the viscous interphase interaction force.

The first phenomenon is obviously the result of the comparatively easier gas "breakthrough" of the segments with porosity which is high as a result of the fluctuations.

The second phenomenon is associated with the fact that the gas which breaks through such segments experiences reduced resistance, and this reduction is not balanced by an increase of the resistance to the gas flowing through the volumes with lowered porosity. Moreover, the porosity fluctuations cause some increase of resistance to regular flow because of the nonlinearity of the function $K(\rho)$ [last term in the parentheses in the second line of (3.1)]. It is easy to see that the influence of this last effect examined previously in [12], is very slight even for high nonlinearity [for example, large n in (2.10)].

If the relation (2.10) is satisfied we have the expression for χ

$$\chi = \left\{1 + n(n+1)\left[\frac{X_7 - X_6}{1-\rho} + \frac{1}{2}\left(\frac{\rho}{1-\rho}\right)^2\left(1-\frac{\rho}{\rho_*}\right)\right]\right\}^{-1}$$

TABLE 3. Dependence of the Quantities $<v'^2>/Q^2$ and $<w'^2>/Q^2$ on ρ and n (first and second numerals in the table groups, respectively)

	$n = 1.0$	1.5	2.0	2.5	3.0	3.5	4.0
0.05	0.0028	0.0051	0.0084	0.0126	0.0177	0.0237	0.0306
	0.0022	0.0035	0.0050	0.0063	0.0089	0.0112	0.0138
0.10	0.0127	0.0230	0.0374	0.0557	0.0779	0.1038	0.1335
	0.0104	0.0161	0.0231	0.0312	0.0404	0.0508	0.0623
0.15	0.0320	0.0578	0.0931	0.1374	0.1965	0.2518	0.3210
	0.0272	0.0419	0.0594	0.0797	0.1025	0.1273	0.1555
0.20	0.0641	0.1142	0.1817	0.2652	0.3633	0.4750	0.5990
	0.0564	0.0857	0.1202	0.1593	0.2027	0.2500	0.3008
0.25	0.1126	0.1974	0.3095	0.4453	0.6019	0.7767	0.9672
	0.1023	0.1533	0.2118	0.2770	0.3478	0.4235	0.5034
0.30	0.1817	0.3128	0.4821	0.6829	0.9094	1.1570	1.4217
	0.1703	0.2509	0.3413	0.4396	0.5441	0.6535	0.7666
0.35	0.2749	0.4645	0.7036	0.9808	1.2869	1.6146	1.9581
	0.2657	0.3848	0.5150	0.6532	0.7969	0.9442	1.0936
0.40	0.3930	0.6525	0.9729	1.3368	1.7311	2.1453	2.5733
	0.3914	0.5579	0.7357	0.9205	1.1089	1.2936	1.4877
0.45	0.5265	0.8634	1.2734	1.7329	2.2244	2.7352	3.2562
	0.5405	0.7619	0.9948	1.2333	1.4734	1.7122	1.9476
0.50	0.6353	1.0428	1.5384	2.0938	2.6383	3.3064	3.9363
	0.6743	0.9509	1.2420	1.5403	1.8407	2.1395	2.4343
0.55	0.5939	1.0001	1.5109	2.1011	2.7511	3.4453	4.1711
	0.6541	0.9453	1.2626	1.5984	1.9466	2.3027	2.6630

Curves of χ (dashed) and $\varphi\chi$ (continuous curves) as a function of ρ for different n are shown in Fig. 3. Both of these coefficients considered as functions of ρ have maxima for $\rho = 0.45 - 0.50$ ($\rho_* = 0.60$). In particular, it is easy to see from Fig. 3 that for n = 3, 50% reduction of the resistance of the system of fluctuating particles in comparison with the system of motionless particles is possible. The coefficients χ and $\chi\varphi$ increase sharply with increase of n. However large values of n are extremely unlikely for locally nonhomogeneous systems since, as a rule, for such n the system is locally homogeneous and the presence analysis is not applicable to this system. The resistance reduction effects were examined previously in [7] qualitatively.

This resistance reduction has been observed repeatedly in fluidized bed experiments and has reached 20-50% (see, for example, the references in [8], and also [13, 14]). In [15] this phenomenon is associated with slow circulation of the suspended material within the bed; in [13, 14] it is explained by the effect of "rectification" of the channels between the particles under conditions of developed fluidization, i.e., in the final analysis by particle pulsations. The second point of view was questioned in [8] on the basis that generation of pseudoturbulence requires additional expenditures of energy of the dispersed medium carrying flow. But in spite of this additional energy consumption, the porosity fluctuation facilitates gas penetration through the segments with reduced particle content to the degree that the total gas flux is still greater than in the analogous system without fluctuations.

We can also assume that for the same reasons the effective resistance of a motionless bed with ordered packing of the particles will differ from the resistance of a motionless bed having disordered packing of the same porosity. Some comparative data confirming this conclusion are presented in [14].

Let us examine in more detail the nonhomogeneity condition (2.12). Using (2.11) and (3.2), we write this condition in the form

$$F = \frac{Q^2}{2ag} > F_0 = \frac{(1-\rho)^2 \chi\varphi^2 X_8}{2c} \left(\frac{2}{3\pi\rho}\right)^{1/3} \left(1 - \frac{\rho}{\rho_*}\right)^{1/3} \tag{3.4}$$

Here F is the Froude number of the flow. The dependence of the critical Froude number F_0, at transition through which the locally nonhomogeneous flow regime is replaced by the locally homogeneous regime, on ρ for n = 1 is shown in Fig. 4 by the continuous curve, and the corresponding dependence of the dimensionless length

$$\frac{L}{L_0} = \frac{1}{k_0 L_0} = \frac{c}{\chi\varphi^2 (1-\rho)^2 X_8}, \qquad L_0 = \frac{Q^2}{g} \tag{3.5}$$

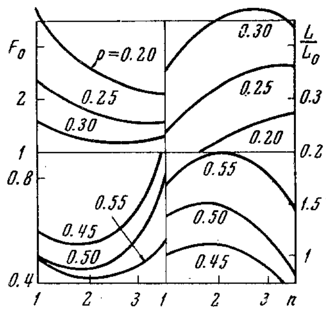

Fig. 5

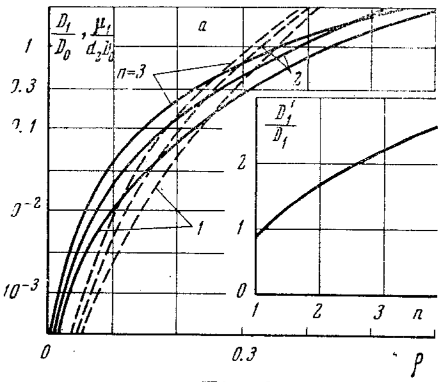

Fig. 6

is shown dashed. It is clear that L (and in some sense L_0 as well) characterizes the spatial scale of the pseudoturbulent motions and can also be considered as the average dimension of the nonhomogeneities which arise under equilibrium conditions. We see from Fig. 4 that L/L_0 increases monotonically (and nearly exponentially) with ρ, reaches a maximum for $\rho \approx 0.55$, and then falls off sharply to zero. The critical Froude number F_0 decreases with increase of ρ, reaches a minimum, and then increases rapidly. For $\rho \to 0$ and $\rho \to \rho_*$ the Froude number $F_0 \to \infty$ and $L/L_0 \to 0$. The curves of L/L_0 and F_0 for other n behave similarly. The dependence of F_0 and L/L_0 on n for several ρ (numerals on curves) is shown in Fig. 5.

The local homogeneity criterion $F < F_0$ actually coincides with the semi-empirical criterion $F \lessgtr 1$, proposed by Wilhelm and Kwauk [16], who studied fluidization of a bed of particles by various dispersed flows medium (see also [14]). Formula (3.5) has the same structure as the expression for the maximal dimension of the stable gas bubble in a fluidized bed, obtained in [8]; however the scale L is several orders less than the diameter of such a stable bubble. We emphasize that the condition (3.4) and the equation (3.5) characterize, naturally, the local properties of the system, which are connected with its structure but in no way characterize the hydrodynamic disturbances which can arise in the flow when stability is disrupted.

Let us examine the monotonic expansion of a fluidized bed of particles with given properties for increasing gas flowrate. Near the initiation of fluidization the bed is locally homogeneous. With further increase of Q two versions are possible: either $F < F_0$ throughout the entire region of existence of the bed and the fluidization is homogeneous for all ρ, or the curves $F(\rho)$ and $F_0(\rho)$ cross for some $\rho = \rho_+$, where we see from Fig. 4 that ρ_+ is very close to ρ_*. Upon passage through ρ_+ the nonhomogeneous regime begins, which is then inevitably again replaced by the homogeneous regime for some $\rho = \rho_-$, which is not necessarily near zero. This pattern of regime replacement is in agreement with the experimental facts [8, 13, 14].

The mean square pseudoturbulent velocities of the phases can be characterized with the aid of the numbers N_V, N_W, introduced above, and the quantities

$$\frac{\langle v'^2\rangle}{Q^2} = \left(\frac{c}{\varphi(1-\rho)}\right)^2 (X_3 + 2X_4), \quad \frac{\langle w'^2\rangle}{Q^2} = \left(\frac{c}{\varphi(1-\rho)}\right)^2 (X_1 + 2X_2)$$

The values of these quantities for different ρ and n are presented in Table 3. We see that for not too small ρ the pseudoturbulent velocities of both phases coincide in order of magnitude with the flux Q, which is in qualitative agreement (as in the dependence of $\langle v'^2 \rangle$ and $\langle w'^2 \rangle$ on ρ) with numerous experimental facts (see bibliography in [13, 14]).

The effective pseudoturbulent pressures (normal stresses) P_j of the dispersed phase in the longitudinal and transverse directions with account for the instantaneous nature of particle momentum transfer in the material is represented in the form [7]

$$\Pi_j = \frac{P_j}{d_2 Q^2} = \rho\left[1 - \left(\frac{\rho}{\rho_*}\right)^{1/3}\right]^{-1} \frac{\langle w_j'^2\rangle}{Q^2} \tag{3.6}$$

The values of Π_j are presented in Table 4. It is not difficult to see that the pressures P_j appearing in the dynamic equations from [2] may have a very marked influence on the motion of the gaseous suspension under nonequilibrium conditions.

TABLE 4. Dependence of Pseudoturbulent Pressures of Dispersed Phase in Longitudinal (Π_1) and Transverse (Π_2) Directions on ρ and n (first and second numerals in table groups, respectively)

	$n = 1.0$	1.5	2.0	2.5	3.0	3.5	4.0
0.05	0.0001	0.0002	0.0003	0.0003	0.0005	0.0006	0.0007
	0.0000	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003
0.10	0.0013	0.0021	0.0030	0.0040	0.0052	0.0066	0.0080
	0.0005	0.0008	0.0011	0.0015	0.0019	0.0024	0.0029
0.15	0.0064	0.0099	0.0140	0.0188	0.0242	0.0301	0.0366
	0.0035	0.0053	0.0076	0.0101	0.0130	0.0163	0.0193
0.20	0.0214	0.0325	0.0456	0.0604	0.0769	0.0948	0.1141
	0.0077	0.0117	0.0164	0.0217	0.0276	0.0341	0.0410
0.25	0.0588	0.0831	0.1218	0.1593	0.2000	0.2435	0.2895
	0.0211	0.0316	0.0437	0.0571	0.0718	0.0873	0.1039
0.30	0.1442	0.2126	0.2892	0.3724	0.4609	0.5536	0.6494
	0.0517	0.0762	0.1036	0.1334	0.1652	0.1934	0.2327
0.35	0.3295	0.4772	0.6387	0.8101	0.9883	1.1709	1.3563
	0.1180	0.1709	0.2287	0.2900	0.3538	0.4193	0.4856
0.40	0.7219	1.0289	1.3569	1.6977	2.0452	2.3950	2.7438
	0.3331	0.4748	0.6262	0.7834	0.9438	1.1052	1.2662
0.45	1.5515	2.1870	2.8555	3.5402	4.2294	4.9147	5.5905
	0.5542	0.7813	1.0201	1.2647	1.5108	1.7557	1.9971
0.50	3.3442	4.7117	6.1541	7.6322	9.1206	10.601	12.062
	1.1884	1.6758	2.1888	2.7145	3.2439	3.7706	4.2901
0.55	7.3874	10.675	14.259	18.051	21.984	26.006	30.074
	2.5939	3.7558	5.0163	6.3504	7.7341	9.1488	10.580

The particle diffusion coefficients are characterized by the ratio α in (2.3), (2.4) and by the quantity

$$\frac{D_1}{D_0} = \frac{0.859 c^2 \rho}{\chi \varphi^3 (1-\rho)^4 X_s}\left(1-\frac{\rho}{\rho_*}\right)^{1/2}, \quad D_0 = \frac{Q^3}{g} \qquad (3.7)$$

The dependence of the dimensionless diffusion coefficient D_1/D_0 in the longitudinal direction on ρ for three values of n is illustrated in Fig. 6a by the continuous curves. The expressions for the momentum and pseudoturbulent energy of the dispersed phase transport coefficients are obtained from the expressions for the corresponding diffusion coefficients with account for the instantaneous nature of particle momentum and energy transfer in the material [7]. As a result we have

$$\frac{\mu_1}{d_2 D_0} = \frac{\lambda_1}{d_2 D_0} = \rho\left[1-\left(\frac{\rho}{\rho_*}\right)^{1/3}\right]^{-1}\frac{D_1}{D_0}, \quad \frac{\mu_2}{d_2 D_0} = \frac{\lambda_2}{d_2 D_0} = \frac{\alpha \mu_1}{d_2 D_0} \qquad (3.8)$$

The quantity μ_1/d_2D_0 is shown dashed in Fig. 6a. As $\rho \to \rho_*$ this quantity becomes infinite and D_1/D_0 approaches some finite value. The latter is associated with use of the assumption of local homogeneity, which is inadequate in the region $\rho \sim \rho_*$. In reality the ratio D_1/D_0 rapidly decreases to zero near ρ_*.

In several problems of heat and mass transport in a gaseous suspension the gas diffusion coefficients may be of interest. We represent them in the form

$$D_1' = \int_0^\infty e^{i\omega\tau}\, d\tau \iint \Psi_{v1,v1}(\omega, \mathbf{k})\, d\omega\, d\mathbf{k}, \quad D_2' = \int_0^\infty e^{i\omega\tau}\, d\tau \iint \Psi_{v2,v2}(\omega, \mathbf{k})\, d\omega\, d\mathbf{k}$$

After calculations we obtain

$$\frac{D_1'}{D_1} = \left[\frac{1}{3} - r^2 - 2\frac{c-1}{c} + \frac{1}{r}\left(r^2 + \frac{c-1}{c}\right)^2 \operatorname{arc\,tg}\frac{1}{r}\right]\left[\frac{1}{3} - r^2 + r^3 \operatorname{arc\,tg}\frac{1}{r}\right]^{-1}, \quad \frac{D_2'}{D_2} \equiv 1 \qquad (3.9)$$

Hence we see that D_1'/D_1 depends on n from (2.10) but is independent of ρ (Fig. 6b). For n which are not too close to one, $D_1' > D_1$. We note that the estimate (3.9) for the gas diffusion coefficients is very approximate, which is associated primarily with the use therein of Eulerian rather than Lagrangian correlations for the gas velocity.

In conclusion we note that the entire analysis developed in section 1 is based on the assumptions that the force of viscous interaction of the gas with a single particle is linear in the relative velocity and can be represented in the Stokes form. The second assumption is not at all essential; dropping this assumption requires only a trivial redefinition of the quantity β in (1.1). The first assumption is incorrect for large Reynolds numbers, when the viscous friction force is quadratic in the velocity. However the calculations remain valid for this case as well if we use the approximate linear approximations for the true interaction force for the different relative velocities.

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